Underwater Vehicle Dynamics

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# Model at a Glance

The 6DOF dynamical equations of motions are given by:

The 6x1 vector is the relative velocity vector for a vehicle in a **steady current**. We have to use relative velocity in order to combine the added mass and inertial matrices into one mass matrix. If the current is accelerating, the equations become much more complicated.

Then

The 6x1 vector describes the hydrostatic loads.

The 6x1 vector gives the applied forces and moments. For our model, these are the wing and tail hydrodynamic forces and moments, the tether tension(s), and any other “propulsive” loads (e.g. rotors/propellers if they exist).

The 6x6 matrix is the mass matrix, which is the sum of the inertial matrix and the added mass matrix.

The 6x6 matric is the combined Corialis-centripetal matrix formed using the relative velocities.

If is formulated such that it does not depend on the linear velocity (as in 3.57 in [1]), then:

The added mass Coriolis-centripetal matrix is formed according to 6.45 and 6.46 in Fossen using the relationship between velocity and relative velocity (i.e. ).

The 6x6 matrix is the damping matrix. See section 6.4 in [1] for more details.

The additional 6 (or 7 if using Euler parameters) kinematical equations of motion are,

and

or

# Derivation

## Reference Frames

There are two reference frames used in the derivation, an inertial reference frame, , and a body-fixed frame, , as described in Table 1.

Table 1. Reference frames.

|  |  |
| --- | --- |
|  | An inertial reference frame which houses the simulation-environment coordinate system. |
|  | A reference frame attached to the vehicle-body. |

A picture containing object

Description automatically generated

Figure 1. Definition of frames.

The origin of the body frame is located at the arbitrary point B. This point may be chosen for convenience. For example, it may be the tether attachment point, it may be the center of mass, or it may be any other point on or off of the body that is geometrically fixed with respect to the body. Likewise, the origin of the inertial reference frame is located at arbitrary point O which may be chosen for convenience. Point F is a point fixed to an arbitrary fluid particle.

## Vectors and Matrices

### Position

Position vectors shown in Table 2 are defined from Figure 1.

Table 2. Position vectors.

|  |  |
| --- | --- |
|  | The position of point B with respect to point O expressed in the frame. |
|  | The position of fluid particle F with respect to the point O expressed in the frame. |
|  | The position of point B with respect to point O expressed in the frame. |
|  | The position to the center of mass of the vehicle from the origin of the body frame expressed in the body frame coordinates. |

### Velocity

The velocity vectors shown in Table 3 are also defined from Figure 1.

Table 3. Velocity vectors.

|  |  |
| --- | --- |
|  | The time derivative of taken in the frame and expressed in the frame. In other words, the inertial velocity of point B expressed in the frame. |
|  | The inertial velocity of fluid particle F expressed in the frame. In a steady, uniform, irrotational current, all fluid particles have the same inertial velocity, thus this vector describes the bulk velocity of the steady uniform flow. |
|  | The inertial velocity of the origin of the body frame relative to the bulk fluid velocity expressed in the body frame. |
|  | The angular velocity of the frame with respect to the frame. |
|  |  |

### Acceleration

Table 4. Acceleration vectors.

|  |  |
| --- | --- |
|  | The time derivative of taken in the frame |
| todo | put the rest of the vectors in this table |

# Dynamics

## Translation Dynamics

Then

Recall . Also,

Then

Finally, from Newton’s second law

This is eqn. 3.32 or 3.33 in Fossen’s handbook.

## Rotation Dynamics

Recall

and

It then follows that

This is equation 3.38 or 3.40 in Fossen’s book.

## Kinematical Equations

### Position

The position of point B is described in the inertial reference frame by:

### Orientation

The orientation kinematics may be expressed using Euler angles or Euler parameters.

#### Euler Angles

For a 3-2-1 body-fixed rotation sequence about angles and , respectively, the rotation matrix is:

From Dr. Mazzoleni’s class we know that:

Solving for and results in:

#### Euler Parameters

The rotation matrix as a function of the Euler parameters is:

Employing we get:

Euler parameters are derived using the assumption that , but the magnitude tends to drift in numerical applications; therefore, the Euler parameters must be normalized regularly. The best normalization method that I have found is the one described in Fossen’s handbook.

The best value for is going to be application specific. Fossen suggests a value of 100 for . I have not come across a case where a value of 100 doesn’t work, and it is the best value in terms of convergence for almost every problem I have encountered.

# States

The orientation states depend on whether Euler angles or Euler parameters are used. The other 9 states are the same for both formulations.

To combine the added mass matrix with the inertia matrix the EOMs must be in terms of the relative velocities. Look at pages 131 (ch6) and 223 (ch8) in Fossen’s handbook and figure out what the states need to be, and how to convert the states into the signals currently used in the simulation.

## Dynamical

The six dynamical states define the velocity of point B and the angular velocity of the body frame with respect to the inertial frame.

Combining these into a single 6x1 state vector:

Then the state derivatives are computed from:

## Kinematical

The position of point B in the inertial reference frame.

Then the derivative is given by:

### Orientation

# State Equations (12 total)

The 6 dynamical state equations are given by 8.147

As an intermediate step, u, v, and w are computed from the relative velocities and the currents speeds. Then the 6 (or 7) kinematical state equations are evaluated. Then the signals in the simulation remain the same, and only the states being integrated change.

# Nomenclature

Todo: list the nomenclature. May be useful later.

# Bibliography

[1] Fossen, T. I. *Handbook of Marine Craft Hydrodynamics and Motion Control*. Wiley, Chichester, West Sussex, 2011.